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by Pardi Affandi

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Optimal Inventory Control System with Stocastic Demand

Pardi Affandi [1*], Faisal [2], Nur Salam [3]

[1, 2, 3] Universitas Lambung Mangkurat, Indonesia

Abstract

Optimal control theory, an extension of the calculus of variations, is a mathematical optimization method with solutions control policy. This method largely inspired by the work of Lev Pontryagin and his colleagues in the Soviet Union and Richard Bellman in the United States. Explicit optimal control is obtained for the two general inventory levels depend inventory production. Inventory is used more specifically limited to the production of inventory problems. The mathematical model of the problem demand inventory can be deterministic and probabilistic or stochastic models. In this research will be discussed how to model the stochastic demand as well as how to solve the inventory model using optimal control techniques.

Keywords: *Inventory Production Problem, Stochastic Demand, Control Theory*

Introduction

Many problems in the life involving systems theory, optimal control theory and some applications. One is the inventory model, the problem is how to manage changes consumer demand in a finished product. So that the company should make good planning in order to produce goods in accordance with the number of requests. The finished goods should fit in a place before it was booked by consumers. This has led to the emergence of inventory that certainly will add to the cost in the form of storage costs such as the cost of physically storing goods or costs arising out of the company's capital tied up in the form of goods. This problem can be modeled using mathematical optimal control techniques.

This study will address the issue of inventory with Model request Stochastic (contains elements of uncertainty) because in the real world turns out request to the inventory tends to change from time to time in accordance with the wishes of consumers, could amount demand lot can also be turned into a bit so that it contains elements of uncertainty.

Literature Review

Inventory Model

Based on demand inventory models are divided into two major groups, namely deterministic and probabilistic. Deterministic model is a model inventory when demand is uncertain, there is a static kind that there is also a dynamic. While the deterministic model is a model inventory when demand is not known with certainty, there is a stationary type that is a probability density function demand remains and there is no stationary ie the probability density function change. The other type is a probabilistic inventory where the demand is always changing, so it tends to form stochastic.

Distribution Poisson and Exponential

In the statistical sciences there are various types of distribution of an event that can be used to analyze the data. Here are some of the statistics contained in the distribution Poisson and Exponential.

An experiment which resulted in the number of successes that occur at time intervals known as Poisson trial. The time interval can be minutes, days, weeks, months, or years. Total success in the experiment called random variable Poisson. Examples of Poisson random variables associated with the time interval is the number of calls per

*All correspondence related to this article should be directed to Pardi Affandi, Universitas Lambung Mangkurat, Indonesia
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hour received an office. Poisson distribution is the probability distribution of the random variable X Poisson who presented the number of successes that occur at specific time intervals. A random variable is said to be Poisson distributed with parameter if the chances density function is given by $f(x) = \frac{e^{-\lambda}\lambda^x}{x!}$, $(x = 0,1,2,3,...)$

The mean and variance of this distribution is given by $E(x) = \lambda$ dan $\sigma_X^2 = \lambda$. (Dimyati & Dimyati, 2002).

Distribution Exponential isused to describe about that eacch of the events distribution occurred with the rate that is constant. That is, the time to serve newcomers do not rely on the longtime that has been spent on airport arrivals before, and does not depend on the number of entrants were waiting to observed. Variables random continuous said to be distributed exponentially with parameter if the function X of the density of the chances given by μ.

$$f(x) = \mu e^{-\mu x}, \quad x \geq 0$$

The function of distribution is given by

$$F(x) = 1 - e^{-\mu x}, \quad x \geq 0$$

The mean and variance of the distribution is given by

$$E[X] = \frac{1}{\mu}, \quad \sigma_X^2 = \frac{1}{\mu^2}$$

Distribution exponential has a meaning that is very important in the analysis of a process of stochastic as having properties *memoryless* ie for any value, t , $s \geq 0$,

$$P(X > t + s | X > t) = P(X > s)$$

If a variable random X is a time of life an equipment then the nature of *memoryless* stated that the chances of equipment are alive least t+S time. If the equipment has been living for *t* h then the equipment that most are not going to live as long as *s* h again. In words other, if the equipment life at time *t* then the distribution of the rest of the time of his life at the distribution time of life actually. it means that the equipment does not remember that the equipment that has been alive for *t* hours, known equipment has been alive for hours is equal to the chance of equipment the life of the early used the bit clock (Ross, 2003).

Process Stochastic

The process of stochastic is the set of variables random $\{X(t), t \in T\}$ that describe $\forall t \in T,\ X(t)$ dynamics of a process. The process of stochastic is a collection random variable, which is a variable $X(t)$ with distribution limited. Generally, index or parameter shows the time t of the process and the variables randomly $X(t)$ show the status of the process at the T time. The set index time of the process called space time (*time space*) and the set of all values of variables random $X(t)$ which may be called the room status (*state space*).

If $T = \{1,2,3,...\}$ can be calculated then the stochastic is discrete and usually expressed with $\{X_t\}$ where $T = \{t | t \geq 0\}$ as calculated, then the stochastic is berparameter continuous and is usually expressed by the notation $\{X(t), t \geq 0\}$. Likewise also the room status, room status is called discrete if finite or not finite can calculated and is called continuous if it contains the interval of the line real (Ghahramani , 2005).

Stochastic Inventory Model

Many phenomena in nature that the emergence / occurrence of irregular both in space and time. To model the evolution of a system containing an uncertainty or a system that runs on an unpredictable environment, where the deterministic model is no longer suitable in use for analyze these systems use Stochastic Processes. Stochastic process is a sequence of events that meet the laws of chance (Karlin & Taylor). Stochastic processes are widely used to model the evolution of a system that includes an uncertainty. Or a system that is operating in an unpredictable environment, where the deterministic model is no longer suitable to analyze the system. This model is suitable for the stated model of inventory on a company by the number of average number of requests in which an element of uncertainty. The element of uncertainty is a role model inventory. The company wants sufficient supply to meet customer demand, but production of too man[illegible]kes could increase the cost and risk of loss through obsolete or property damage. The production of too little increases the risk of lost sales and lost customers as a result. Resource managers must set the number of items stored in the level that balances demand risk and the risk of shortages. The operations manager must be able to adjust the master production schedule considering the precise nature of [illegible] estimated future demand and lead time must be from the manufacturing process. this situation is common, and the answers obtained from deterministic analysis is very often unsatisfactory. So the demand for stochastic models and models presented provide an appropriate response

Optimal control

Definisi (M.Athans 1966) *Optimal control problem for the system with the target S, the objective function* $J(x_0,t_0,u)$*, the set of admissible control U, and the initial state* x_0 *at time* t_0 *is decisive control* $u \in U$ *that maximizes objective function* $J(u)$*. Any control u * which provides a solution to the problem of optimal control called optimal control*.

In the following discussion, the problems given in the case of optimal control with state end and the end time is known. In other words, the target set S shaped S = $\{x_1\}$x$\{t_1\}$ in the form (x_1,t_1) with x_1 specially element in R^n and t_1 element at (T_1,T_2).

Given the state system by the end and the end time unknown

$$\dot{x}(t) = f[x(t),u(t),t]$$

with x(t) vector state sized $nx1$, u(t) input vector sized $mx1$, f a vector valued function. Initially given state is X_0 and initially time is t_0. Target set S form (x_1,t_1) with $t_1 \in (T_1,T_2)$ known value and $t_1 > t_0$.

Optimal control problem is to find the admissible control u(t) with the initial value (x_0,t_0) and the final value (x_1,t_1) that maximizes the objective function $J(u) = \int_{t_0}^{t_1} L(x(t),u(t),t)\ dt$

To solve the problems mentioned above optimal control, first determined necessary condition for optimal control are met.

Discussion Probabilistic Inventory Models

The previous model is a deterministic model in which all parameters are known with certainty. In fact often the case these parameters are values that are uncertain as the annual demand, daily demand, lead time, cost savings, the cost of the message, the cost of running out and prices. Stochastic or probabilistic processes in the inventory system will constantly come across in a state of fact. Demand is happening is not always deterministic. There are times when demand or request an item on the company's varied or probabilistic distributions follow certain set of characteristics known. To cope with varying demand companies usually have certain supplies as a safety-called *Safety / buffer Stock* . *Safety stock* provides a number of inventory during the *lead time* . To resolve such questions used approach is probabilistic inventory. The model used in the completion of a probabilistic demand include continuous inventory system (System Q, r) and the periodic inventory system (System P).

The assumptions used in the model system of continuous inventory *Q, R* and System *P* are:

1. Distribution of demand for goods known opportunities,

2. Time square constant bookings,

3. The price of the goods ordered constant, not depending on the size of the booking.

4. The booking fee for each booking constant,

5. The cost of inventory shortages expressed in cost per unit regardless shortage shortage ever took place.

6. Cost savings per year per unit is constant, not depending on the number of items in the store.

Results And Discussion

Probabilistic Inventory Models

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Stochastic model supplies the most widely used is the system *Q, r* which is also called the system checks continuously, system reorder point, and the booking system fixed. By using the system *Q, r* whenever held smear collection, the amount of preparation was left to be counted for determine whether reordering as yet been necessary.

Rules of this model is the book back when the position of the preparation has been similar to or smaller than the reorder point. In this model, the amount of each order (*Q*) is the same from time to time, but the interval between two successive bookings are fickle. Besides, the waiting period (*lead time*) is the same for each period are different.

In the system model *P* supplies checked regularly (*periodically*) every certain period and this period does not change over time. Reordering is done to the order (*R*) which varies but with a fixed interval between two consecutive orders. Due to this fixed intervals, as well as checks are conducted regularly, then the system *P* is also called the system of periodic inspection, periodic order system, ordering system with a fixed distance or booking system back periodically. In the system *P* , was designated an inventory target is the level of inventory that must be achieved each time the reservation is made.

The element of uncertainty is very instrumental in the model inventory . The company wants sufficient supply to meet customer demand, but production too many hikes may increase the cost and risk of loss by going to usangan or damage to the goods . The production of too little increases the risk of losing sales and consequently lost customers . Managers sources the power should regulate the number ofitems stored in the level that balances the risk of demand and the risk of shortages . S state all is common, and the answers obtained from the analysis of deterministic very often not satisfactory for user model. So the demand model will air the nature of the stochastic and the model presented gives answers accordingly. One element of uncertainty considered in this section is the demand for the product from inventory. In assuming the demand is not known, but that the probability distribution of the demand note. Mathematical derivation will determine policy optimal in terms of distribution. Some of the terms used in the term stochastic demand:

Reorder point (reorder point) is an inventory level that remains in stock an amount equal to the demand during the period of time required to receive orders (called *lead time*).

Variable random on Demand (x) : It is a variable random that the demand for a specific time. Care should be taken to identify the period of variable random defined as different between models whereconsidered.

Probability Demand Discrete *the distribution function P (x)* : When the demand is assumed variables discrete random , P(x) gives the probability that the demand is equal to x.

Functions Distribution Cumulative Dis Krit F (b) : The probability of a request that is less than or equal to b is F (b) when the demand is discrete . $F(b) = \sum_{x=0}^{b} P(x)$.

Probability Demand Continuous density function f (x) : When we know the demand assumed continuous, *f(x)* is a function of its density . Then the probability of demand is between a and b so that $P(a \leq X \leq b) = \int_a^b f(x)dx$. On the assumption right that the demand is nonnegative, so that f (x) is zero for the values of the negative .

Standard Normal Distribution Function f (x) and φ (x) : This is a function of the density and the cumulative distribution function for a standard normal distribution.

Furthermore, the term probability distribution function or probability density function known as pdf, while the cumulative distribution function as CDF.

In modeling stochastic including important decision in determining the distribution of which is used for the request. A common assumption is that events demand is independent. This assumption leads to the Poisson distribution when demand is expected in a small time interval and the normal distribution when demand is expected great. Next will continue into the average level of demand. in the interval t demand is expected in the Poisson distribution form $P(x) = \frac{(at)^x e^{-(at)}}{x!}$.

When in great demand Poisson distribution can be approximated by a normal distribution with a mean and standard deviation of μ = at, and $\tau = \sqrt{at}$. Value *F (b)* evaluated using the table for the standard normal distribution. Of course the distribution of another can be assumed to request other forms. The general assumption is a normal distribution with other values of the mean and standard deviation, uniform distribution, and the exponential distribution. The last two are useful for the analysis much simpler.

Finding and Advantages Disadvantages expected Expected

Concerns about demand relationship for some time period relative to the level of inventory at the beginning of the period of time is one of the natural thing. If the demand is less than the level of opening stock, no inventory remaining at the end of the interval. It is a condition of excess. If the demand is greater than the initial inventory levels, will have the condition of shortage.

At some point, consider the level of inventories is a positive z value. For some interval of time, the demand is a random variable x with pdf, f(x), and the CDF, F(x). Mean and standard deviation of this distribution is and respectively. With a given distribution, while the probability of shortage denoted Ps, and the probability of excess with Pe. For a continuous distribution is obtained

$$Ps = P\{x > z\} = \int_z^\infty f(x)dx = 1 - F(z)$$

$$Pe = P\{x \leq z\} = \int_z^\infty f(x)\,dx = F(z)$$

In some cases the problem shortage is expected denoted by *Es* . It depends on whether the demand is greater or less than z. Thus obtained:

$$\text{Disadvantages objects} \begin{cases} 0, & jika\ x \leq z \\ x - z, & jika\ x > z \end{cases}$$

Then the Es is expected short

$$\mathrm{Es} = \int_{z}^{\infty} (x - z) f(x) dx$$

Like wise for the surplus, the excess is hoped *Ee* will be obtained values of

Expected expressed in Ee Thus obtained

$$\mathrm{Ee} = \int_0^{z} (z - x) f(x) dx = z - \mu + \mathrm{Es}$$

For discrete distributions, the notation integral in the press amaan be sigma amount so obtained

$$\mathrm{Ps} = \mathrm{P}\{x \geq z\} = \sum_{z}^{\infty} \mathrm{P}(x) dx = 1 - \mathrm{F}(z)$$

$$\mathrm{Pe} = \mathrm{P}\{x \leq z\} = \sum_{0}^{z} \mathrm{P}(x) dx = \mathrm{F}(z)$$

$$\mathrm{Es} = \sum_{z}^{\infty} (x - z) \mathrm{P}(x) dx = 1 - \mathrm{F}(z)$$

$$\mathrm{Ee} = \sum_{0}^{z} (z - x) \mathrm{P}(x) dx = \mathrm{F}(z) = z - \mu + \mathrm{Es}$$

Distribution Demand is Normal

When demand during lead time has a normal distribution, the table is used to find the number. Assume demand during lead time has a normal distribution with mean μ and standard deviation σ . Then it will dit entukan inventory level in terms of this number of standard deviations away from the mean. $Z = \mu + k\sigma$ or $k = \frac{z - \mu}{\sigma}$.

Before have identified two things that the first of these functions as a pdf and CDF next is defined as

$$G(k) = \int_k^{\infty} (y - k)\phi(y) dy = \phi(k) - k[1 - \Phi(k)]$$

With m used relationship between the standard normal distribution and normal, will be obtained by the following relationship $f(z) = \frac{1}{\sigma}\phi(k)\ and\ F(z) = \Phi(k)$

$$\mathrm{Es}(z) = \sigma G(k)$$

$$\mathrm{Ee} = z - \mu + \sigma G(k)$$

Inventory Stochastic Single Period

This policy model is similar to the policy *c ontinous review* , the difference in this policy, taking inventory done only once (inventory reduction occurs only once), and when the inventory level reaches*the reorder level* , then made bookings by Q. In this policy, the variable Q and r must be determined to achieve minimal total cost of inventory. This policy model, especially applied to two types , namely the first inquiry of the items on rare intervals which type these requests for items that follow the model of rapid change, the needs are rarely damaged components and spare parts specific items for maintenance and repair. The second request is uncertain for the short-lived item at frequent intervals. A request like this, especially for items that expire quickly (newspapers, weekly magazines, Christmas cards).

Items with a single booking has a pattern of demand with the use of limited sales period. The item ordered either from external suppliers or own production at the beginning of the period, and there is no chance for a second booking during this period. If demand for the period is greater than the amount that has been booked, it will lose profits.

This section considers the situation of the inventory in which the order for replenishment inventory can be evaluated independently of future decisions. This case occurs when the supply can not be added later (parts for space travel), or when conditions become obsolete inventories (fresh fruit, this time the newspaper). The problem has several periods, but a decision on the current inventory must be independent of the period ahead. In assuming no charge for placing the order filling, and then in assuming that there is a charge. Period single model with no arrangement fee and consideration inventory situation in which the trader must buy a quantity of goods offered for sale during the time interval specified . Suppose b charcoal-goods purchased for the cost *c* per unit and sold at a price *b* each unit. If an item remains unsold at the end of the period, inventories have rescue with a value specified . If the demand is not satisfied during the interval specified , there is a charge of *d* per unit of shortcomings. Demand during the period is a random x variable with pdf and CDF given her . The problem is to determine the number of items to be purchased. In this case Is right as the order level S, for the purchase bring supplies to the level of S. For this part, there is no fee for placing an order for the item. Expression of profits during the interval depends on whether the request falls above or below S. If the demand is less than S, Revenue earned only to the number sold x, while the quantity purchased is obtained for S. Inventories of unsold quantity S - x. The advantage in this case is sold S. Thus advantage.

Advantage = $bx - cS + a\ (S - x)$ for x member of the S .

If the demand is greater than S, revenues are only obtained for the sale of S. A shortage costs *d* be issued for each something brief item of x - S , the advantage in this case will be obtained,

Advantage = $BS - cS - d\ (x - S)$ for every x member S .

Assuming a continuous distribution and take the expectation of all the values of the random variable, the expected profit is

$$E[\text{Profit}] = b\int_0^S xf(x)dx + b\int_S^\alpha Sf(x)dx - cS + a\int_0^S (S-x)f(x)dx - a\int_S^\alpha (x-S)f(x)dx$$

Can be simplified into

$$E[\text{Profit}] = b\mu - cS + a\int_0^S (S-x)f(x)dx - a\int_S^\alpha (x-S)f(x)dx$$

Expected value of *Ee* will be excessive so, l aba written in this case as

E [Profit] = $b - cS + AEE - (d + b)\ Es$

To find the level of order optimal, will dite set apart derivative of profits with a requirement for S equal to zero. Thus obtained

$$\frac{dE[Profit]}{dS} = -c + a\int_0^S f(x)dx + (d+b)\int_S^\alpha f(x)dx = 0$$

Or that the value of

$$\frac{dE[Profit]}{dS} = -c + aF(s) + (d+b)\big(1 - F(s)\big) = 0$$

CDF of the level of order optimal S * defined by

$$F(S^*) = \frac{b - c + d}{b - a + d}$$

This result is sometimes expressed in the purchase cost

c, the cost of storage h, issued for each unit shortfall at the end of the period. D nature of this case the optimal expected cost is

E [Cost] = $cS + Hee + PES$.

Then the optimal solution is obtained

$$F(S^*) = \frac{p - c}{p + h}$$

Two similar solutions if carried identification h = -a = negative of the residual

value of p = b + d = loss of income per unit + lack of funds.

If demand during this period had a normal distribution with a mean and standard deviation and the expected profit is evaluated for each level of the value given. The level of orders expressed in of the number of standard deviations from the mean, or

$S = \mu + \sigma k$.

Optimality condition becomes

$$\Phi(S^*) = \frac{b - c + d}{b - a + d} = \frac{p - c}{p + h}$$

The expected value profit is evaluated by expression

E[Profit] = $b\mu - cS + a[S - \mu + \sigma G(k)] - (d + b)\sigma G(k)$.

K uantitas value on the right side of the equation above as the value of the threshold. Optimality condition to the level of the order value for the CDF. For a continuous random variable is no solution if the value of the threshold ranges from 0 to 1. N use values reasonable parameters will generate the threshold is less than 0 or greater than 1. For optimal value of the discrete distribution rate order is the smallest value of S so

E [Profit | $S + 1$] £ E [Profit | $S + 1$].

By manipulation of the term summation to determine the benefits expected, may indicate that the optimal sequence level is the smallest value of S CDF equal to or exceeds the threshold. The value is

$$F(S^*) \geq \frac{b - c + d}{b - a + d} \, atau \, \frac{p - c}{p + h}$$

Optimal Control with Stochastic Demand

The state of the system is represented by a model of control optimally to the shape p roses stochastic, but a right in consider differential equations of the type known as the equation through the process of diffusion Markov, and the goal of his was to synthesize feedback with optimal control for systems of equations.

In the next section will be introduced in the model decisions that consider both the risks and risk-free. Stochastic optimal control problems involving Markov processes, which were introduced by Sethi and Zhang (1994a, 1994c). It will use the variable state to be the thing observed, and use Framework Jacobi - Bellman model of stochastic principle of maximum.

Maximize

$$E[\int_0^T F(X_t,U_t,t)dt + S(X_T,T)], \qquad (13.28)$$

Based on stochastic differential equations

$$dX_t = f(X_t,U_t,t)dt + G(X_t,U_t,t)dz_t, \quad X_0 = x_0. \qquad (4.2)$$

V (x, t) which fulfill the function value

$$V(x,t) = \max_u E[F(x,u,t)dt + V(x+dX_t,t$$

With the expansion of Taylor, obtained

$$\begin{aligned} V(x+dX_t,t+dt) = V(x,t) & + V_t dt + V_x dX_t + \tfrac{1}{2}V_{xx}(dX_t)^2 \\ & + \tfrac{1}{2}V_{tt}(dt)^2 + \tfrac{1}{2}V_{xt}dX_t dt \\ & + \text{higher-order terms.} \end{aligned} \qquad (4.4)$$

From (4.1), can be written that

$$\begin{aligned} (dX_t)^2 &= f^2(dt)^2 + G^2(dz_t)^2 + 2fGdz_t \\ dX_t dt &= f(dt)^2 + Gdz_t dt. \end{aligned} \qquad (4.5)$$

Rules multiplication of stochastic calculus is:

$$(dz_t)^2 = dt, \; dz_t.dt = 0, \; dt^2 = 0. \qquad (4.6)$$

$$V = \max_u E\left[Fdt + V + V_t dt + V_x f dt + \frac{1}{2}V_{xx}G^2 dt\right] \qquad (4.7)$$

$$0 = \max_u [F + V_t + V_x f + \frac{1}{2}V_{xx}G^2] \qquad (4.8)$$

$$V(x,T) = S(x,T). \qquad (4.9)$$

The notations used in Model Stock stochastic is as follows:

X_t = the level of inventory at time t (state variables),

Ut = the production rate at time t (variable control),

S = rate constant demand at time t; S> 0,

T = length of the planning period,

× = The rate of factory inventories - optimal,

û = level of factory production - optimal,

x_0 = initial inventory levels,

h = coefficient of inventory storage fees,

c = coefficient of production costs,

B = the residual value per unit of inventory at the time T,

z_t = Wiener process standards,

σ = diffusion coefficient constant.

A problem of inventory model stochastic obtained

$$dX_t = (U_t - S)dt + \sigma dz_t, \ X_0 = x_0, \quad (4.10)$$

$$\min E\left\{\int_0^T [c(U_t - \bar{u})^2 + h(X_t - \quad (4.11)$$

$$\bar{x} = \bar{u} = 0 \ \text{and} \ h = c = \quad (4.12)$$

$$\max E\left\{\int_0^T -(U_t^2 + X_t^2)dt + \quad (4.13)$$

With V (x, t) is a function of the value specified. This meets the shape of the Hamilton-Jacobi-Bellman (HJB) equation

$$0 = \max_u[-(u^2 + x^2) + V_t + V_x(u - S) + \frac{1}{2}\sigma \quad (4.14)$$

$$V(x,T) = Bx. \quad (4.15)$$

$$u(x,t) = \frac{V_x(x,t)}{2}. \quad (4.16)$$

$$0 = \frac{V_x^2}{4} - x^2 + V_t - SV_x + \frac{1}{2}\sigma^2 \quad (4.17)$$

From equation 4 .1 , if the level of production will be limited to non-negative, then (4:15) will be converted into equation

$$u(x,t) = \max\left[0, \frac{V_x(x,t)}{2}\right]. \quad (4.18)$$

Solution to Optimal Control with Stochastic Demand

$$V(x,t) = Q(t)x^2 + R(t) + M(t). \quad (4.19)$$

$$V_t = \dot{Q}x^2 + \dot{R}x + \dot{M}, \tag{4.20}$$

$$V_x = 2Qx + R, \tag{4.21}$$

$$V_{xx} = 2Q, \tag{4.22}$$

$$x^2[\dot{Q} + Q^2 - 1] + x[\dot{R} + RQ - 2SQ] + \dot{M} + \frac{R^2}{2} - RS + \sigma^2\zeta \quad (4.23) \quad 51)$$

From equation (4 , 23) applies to any value of x, must have a solution

$$\begin{aligned} \dot{Q} &= 1 - Q^2, \quad Q(T) = 0, \\ \dot{R} &= 2SQ - RQ, \quad R(T) = B, \\ \dot{M} &= RS - \frac{R^2}{4} - \sigma^2 Q, \quad M(T) = 0, \end{aligned} \quad (4.24)$$

where the boundary condition is obtained by comparing (4:19) with the boundary condition V (x, T) = Bx of equation (4:15).

To complete the equation (4:24), will be on extending the partial fractions to get a new equation means obtained

$$\frac{\dot{Q}}{2}\left[\frac{1}{1-Q} + \frac{1}{1+Q}\right] = 1, \tag{4.25}$$

$$Q = \frac{y-1}{y+1}, \tag{4.26}$$

where the value

$$y = e^{2(t-T)}. \quad (4.27) \quad (10.56)$$

Because S is assumed to be constant, will be able to reduce (4:24) so that

$$\dot{R}^0 + R^0 Q = 0, \quad R^0(T) = B - 2S.$$

With the change of variables defined by the solutions provided by $\log R^0(T) - \log R^0(t) = -\int_t^T Q(\tau)d\tau,$

It will obtain the value of the desired result is

$$R = 2S + \frac{2(B-2S)\sqrt{y}}{y+1}. \tag{4.28}$$

$$M(t) = -\int_t^T [RS - R^2/4 - \sigma$$

$$u^* = V_x/2 = Qx + R/2 = S + \frac{(y-1)x + (B-2S),}{y+1} \tag{4.29}$$

From equation (4.2) The production level in the optimal (4:29) at the request plus correction term that depends on inventory levels and within leveled during the time T. Because the $(y-1) < 0$ for $t < T$, it is clear that for value- a lower value of x, the optimal production will be obtained tend to be positive. However, if x is very high, a correction term will be smaller than S, and optimal control will be negative. In other words, if inventory levels are too high, the plant can save money by removing part of the inventory resulting in lower costs there, it can be seen quite clearly in Sethi and Zhang (1994a) and Ying and Zhang (1997).

Conclusion

Control theory can be used to solve inventory problems with stochastic production. This model can be extended in various ways. For example, instead of minimizing the total cost, but want to maximize total production units or you can also model form inventory with stochastic demand.

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