Optimal-Control-For-Dysentery-Epidemic-Model-With-Treatment

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Abstract: This paper aims to made a model and analyze the a dysentery diarrhea epidemic using a SIR-T (Susceptible-Infected-Recovered and Treatment) From the model obtained, be analyzed the stability criteria around the disease-free equilibrium point. Next, perform Optimal Control used treatments. Furthermore, by involving The Pontryagin's maximum principle to complete the Mathematical Model of dysentery diarrhea epidemic obtained.

Index Terms: SIR-T model, dysentery, Optimal Control

1 Introduction

DYSENTERY diarrhea epidemic is spread by fecal contamination of food and water, usually in impoverished areas with poor sanitation. Someone said to suffer from diarrhea when the stool is more runny than usual, or when defecating three or more times, or defecating watery but not bleeding within 24 hours [6]. According to the results of the Basic Health Research [4] showed that Dysentery diarrhea disease is the main cause of death in infants. Likewise, the data obtained from the Head of the South Kalimantan Health Office of Banjarmasin City, Dr. Diah R Praswati in August 2018, recorded that there were 924 people suffering from diarrhea. This number increased from July to 839 people. However, this number is still lower than before in January 2018, there were 1,067 diarrhea patients recorded, then in February 2018 the number was 944, March 2018 were 946, April 2018 were 980, May 2018 were 1,011, June 2018 were 992 and July 2018 were 839 "This data is taken from 26 health centers in the city of Banjarmasin. The eradication of diarrheal diseases is always carried out by the government through the South Kalimantan Health Service (2016) and their achievements have improved, especially after the dry season, but it is still a problem and need to make even harder efforts to achieve the target of diarrhea free of 2025. So in addition to treatment, also needed the right treatment, treatment is a step taken to overcome existing problems. In addition, another alternative to eradicating disease is to form a model to further control or control it.

A model is a general characteristic that represents a group of existing forms or representations of a problem in a simpler and easier to work form. In mathematics, model theory is the science that presents the concepts of sets or the knowledge of models that support a systematic system. Model theory begins with the assumption of the existence of mathematical objects such as the existence of all numbers and then looks for the existence of operations, relations, or axioms attached to each object or those objects. Mathematical models obtained from a given mathematical problem, then solved by the rules of the existing rules. Settlement obtained, needs to be tested to determine whether the settlement is valid or not. A valid result will correctly answer the mathematical model and is called a mathematical solution. If the solution is invalid or does not meet the mathematical model, then the solution to the problem has not been found, and need to solve the mathematical model [5]. The control can be carried out precise mathematical modeling, L Ross (1911) first modeled the spread of disease using mathematical models. Kermack & McKendrick (1927) continued to introduce one of the basic epidemic models, SIR. This model consists of three compartments namely Sucseptible (vulnerable), Infected (infection), and Recovered (cured). Therefore, it is necessary to have an action to reduce the rate of spread of diarrheal diseases, one of which is to know the pattern of spread of diarrheal disease. Mathematics can be used to determine the spread pattern of diarrheal diseases by utilizing the SIR-T mathematical model. Mathemathical SIR model and SEIR model research by P Affandi and collegues [1][2]. Research on the model of the spread of diarrheal disease has been widely carried out. One of them is modeling the spread of diarrheal disease as a result of research by Ojaswita Chaturvedi and colleagues [3]. The research formed a SIR mathematical model with a case study of the spread of Dysentery diarrhea disease with one population, because it only examined one population then in this model the death rate of diarrheal disease was not considered. S.O. Adewale and colleagues (2015) also maked research about model considered four (4) compartmental models to gain insight into the effect of vaccine on the dynamical spread of diarrhea disease in a community [7]. In this study, the authors examined how the spread of Dysentery diarrhea through disease models of the SIR-T (Susceptible-Infected-Recovered with Treatment) mathematical models will analyze the characteristics of the spread of diarrheal disease. The SIR-T population model is a mathematical model to describe a disease involving treatments where an infected patient can recover and get the disease again. The initial step is to determine the parameters that most influence Dysentery diarrhea disease. Furthermore, controlling involves involving treatment of individuals, and analyzing the level of Dysentery diarrhea infection.

2 Methodology

The research is planned to begin by conducting a literature review in the form of research results that reveal the formation of dysentery diarrhea disease models, especially the factors and assumptions of the models that have been carried out by several previous researchers. The results of the study form the basis for developing initial assumptions

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and determining initial models that can be implemented and collecting secondary data through survey data through an appropriate, reliable and valid data questionnaire. Stages of processing research data using steps in the statistical method, beginning with determining the parameters that are most influential as a factor causing diarrheal disease. This is done with the help of the regression method so that from several factors a dominant factor will be obtained. The next step, the most influential parameters will be involved in the stage of establishing the diarrhea disease model, then conducting control in the form of a vaccine and treatment to complete the mathematical model of the spread of diarrheal disease.

3 Model Formulation

We consider an SIR-T model. The model considered for (4) compartmental models to gain insight into the treatment on the dynamical spread of diarrhea disease in a community. Assumptions used in the diarrheal disease model population of toddlers and adults whose natural birth and death rates are considered constant, population is homogeneous, meaning that every individual has the same chance of developing diarrhea, transmission of diarrheal disease only through direct contact with patient feces, only one disease in the population, individuals born from adult susceptible classes and infected adults will become individuals who are susceptible to diarrheal disease, and infected individuals can recover from diarrheal disease.

3.1 Model Diagram

This model subdivides SIR-T model. Thus, at time t, the population:

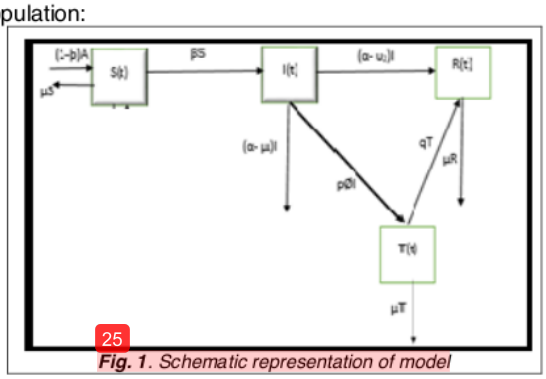

Fig. 1. Schematic representation of model

TABLE 5

DESCRIPTION OF VARIABLES AND PARAMETERS OF THE MODEL.

Parameters	Description
S(t)	the number of susceptible individuals at time t
I(t)	the number of infected individuals at time t
R(t)	the number of recovered at time t
T(t)	treated class at time t
þ	class of susceptible recruited individuals to be vaccinated
α	constant recovery rate
β	rate of contact that is sufficient to transmit the disease
b	rate at which the vaccine wears off s
d	death rate due to diarrhea disease
μ	of natural death.
u_1	rate at which the susceptible population is vaccinated
u_2	rate at which infected people are treated
A	Recruitment rate by birth or immigration
τ	rate at which infected people are recovered

The population of susceptible individuals further reduced by natural death at the rate μ. We have

$$\frac{dS}{dt} = (1-þ)A - \beta SI(t) + bV(t) - u_1 S(t) - \mu S(t)$$

The class population infected diarrhea individual increases by the susceptible individuals the rate β the population later decreased by treatment rate (τ) for diarrhea infected individual and finally reduced by the natural death rate, induced mortality death rate at μ. Thus,

$$\frac{dI}{dt} = \beta SI(t) + (\alpha+\mu)I(t) - (\alpha+u_2)I(t) - p\emptyset I(t)$$

Recovered individuals are those that is further denoted by the number of individuals who recover from diarrhea at the time t. The population increased by new recovered from infected individuals who acquire diarrhea infection with effective contact with people infected with diarrhea, the population reduced by natural death μ and progression from recovered class to recovered class at the rate might also from treatment. Hence,

$$\frac{dR}{dt} = (\alpha+u_2)I(t) + \tau T(t) - \mu R(t)$$

Individuals the treated class at time t denotes with,

$$\frac{dT}{dt} = p\emptyset I(t) - \tau T(t) - \mu I(t)$$

3.2 Analysis Model

From the description of the dynamics of diarrhea as depicted in Figure 1, we have the following set of ordinary differential equations system.

$$\frac{dS}{dt} = (1-þ)A - \beta SI(t) + bV(t) - u_1 S(t) - \mu S(t)$$

$$\frac{dI}{dt} = \beta SI(t) + (\alpha+\mu)I(t) - (\alpha+u_2)I(t) - p\emptyset I(t)$$

$$\frac{dR}{dt} = (\alpha+u_2)I(t) + \tau T(t) - \mu R(t)$$

$$\frac{dT}{dt} = p\emptyset I(t) - \tau T(t) - \mu I(t)$$

with initial conditions $S(0) = S_0$, $I(0) = I_0$, $R(0) = R_0$, $T(0) = T_0$.

3.2.1 Existence and Uniqueness of solution

Theorem 1: Following (Derrick and Grossman 1976) Let

$$x_1^1 = f_1(x_1, x_2, x_3, \dots, t), x_1(t_0) = x_{1A}$$

$$x_2^1 = f_1(x_1, x_2, x_3, \dots, t), x_2(t_0) = x_{2A}$$

$$x_3^1 = f_3(x_1, x_2, x_3, \dots, t), x_3(t_0) = x_{3A}$$

$$\vdots$$

$$x_n^1 = f_n(x_1, x_2, x_3, \dots, t), x_n(t_0) = x_{nA}$$

Let D denote the region in [(n+1)-dimensional space one dimension for t and n dimensions for the vector x]. If the partial derivatives $\frac{\partial fi}{\partial xj}$, i, j = 1,2,..., n are continuous D =

{(x,t),/t-t_0/≤a, x-x_0/≤b }. Then there is a constant δ >0 such that there exists a unique continuous vector solution $\underline{x} = [x_1(t), x_2(t), x_3(t), \dots, x_n(t)]$

Theorem 2.

Let

$$\frac{dS}{dt} = f_1 = (1-þ)A - \beta SI(t) + bV(t) - u_1 S(t) - \mu S(t)$$
$$\frac{dI}{dt} = f_2 = \beta SI(t) + (\alpha+\mu)I(t) - (\alpha+u_2)I(t) - p\emptyset I(t)$$
$$\frac{dR}{dt} = f_3 = (\alpha+u_2)I(t) + \tau T(t) - \mu R(t)$$
$$\frac{dT}{dt} = f_5 = p\emptyset I(t) - \tau T(t) - \mu I(t)$$

with initial conditions $S(t_0) = S_0$, $I(t_0) = I_0$, $R(t_0) = R_0$, $T(t_0) = T_0$.

D = {(S,I,R,T),/S-S_0/≤ a, /I-I_0/≤ b, R-R_0/≤ c, /T-T_0/≤ e }

Then equation has a unique solution.

Proof:

For

$$f_1(S,I,R,V,T) = (1-þ)A - \beta SI(t) + bV(t) - u_1 S(t) - \mu S(t)$$
$$\frac{\partial f_1}{\partial S} = -\beta I(t) - u_1 - \mu$$
$$\frac{\partial f_1}{\partial I} = -\beta S(t) \qquad \frac{\partial f_1}{\partial R} = 0$$
$$\frac{\partial f_1}{\partial V} = b \qquad \frac{\partial f_1}{\partial T} = 0$$

$$f_2(S,I,R,T) = \beta SI(t) + (\alpha+\mu)I(t) - (\alpha+u_2)I(t) - p\emptyset I(t)$$
$$\frac{\partial f_2}{\partial S} = \beta I(t)$$
$$\frac{\partial f_2}{\partial I} = -\beta S(t) \qquad \frac{\partial f_2}{\partial R} = 0$$
$$\frac{\partial f_2}{\partial T} = 0$$

$$f_3(S,I,R,T) = (\alpha+u_2)I(t) + \tau T(t) - \mu R(t)$$
$$\frac{\partial f_3}{\partial S} = 0$$
$$\frac{\partial f_3}{\partial I} = \alpha + u_2 \qquad \frac{\partial f_3}{\partial R} = -\mu$$
$$\frac{\partial f_3}{\partial V} = 0 \qquad \frac{\partial f_3}{\partial T} = \tau$$

$$f_4(S,I,R,T) = þA + u_1 S(t) - bVT(t) - \mu V(t)$$
$$\frac{\partial f_4}{\partial S} = u_1$$
$$\frac{\partial f_4}{\partial I} = 0 \qquad \frac{\partial f_4}{\partial R} = 0$$
$$\frac{\partial f_4}{\partial V} = -bT(t) \qquad \frac{\partial f_4}{\partial T} = -bV$$

$$f_5(S,I,R,T) = p\emptyset I(t) - \tau T(t) - \mu I(t)$$
$$\frac{\partial f_5}{\partial S} = \beta I(t)$$
$$\frac{\partial f_5}{\partial I} = -\beta S(t) \qquad \frac{\partial f_5}{\partial R} = 0$$
$$\frac{\partial f_5}{\partial V} = 0 \qquad \frac{\partial f_5}{\partial T} = 0$$

Hence the problem has taken a unique solution and the model is mathematically and epidemiologically well posed.

3.3 Calculations and analysis

The point of free equilibrium is the state at which there are no infections in the population. For the population to be deprived of the pathogens, the infected states will be assumed to be zero The stability of an equilibrium point can be determined based on the eigenvalues matrix. System we have with model nonlinear differential equations it can be determined by linearizing using a Jacobian matrix. This Dysentery diarrhea epidemic model has two equilibrium points, namely the disease-free equilibrium point and the endemic equilibrium point. By definition, equilibrium point equations are derived from $\frac{dS}{dt} = 0, \frac{dI}{dt} = 0, \frac{dR}{dt} = 0, \frac{dV}{dt} = 0$, $\frac{dT}{dt} = 0$. To determine the equilibrium point, the equation is formed into, $\frac{dI}{dt} = 0$. Therefore I = 0. At this state, the only non-zero class is the susceptible class. so we have obtained value $S^* = \frac{\mu - u_2}{\beta}$ order to get the asymptotic state, the right hand side of equation will be equated to zero.

The point of disease-free equilibrium is a condition in which no more illness attacks or no more infected individuals if I =0 substituted to the previous equation then obtained: S = $\frac{\mu - u_2}{\beta}$. Then if I=0 substituted to the previous equation : R=0 So that obtained point of equilibrium free of disease that is

$E_0 = (\frac{\mu - u_2}{\beta}, 0, 0, 0)$.

The endemic equilibrium point is a condition in which the invading disease still exists and is still spreading if I ≠0 then we have

$E_1 = (\frac{p(\mu - u_2)}{\beta}$, (α+d+b)/β, (B + K)β-μ(α+d+L)/β(α+d+L), (B + K)βα-μα(α+d+p), βμ(α+d+L)).

3.4 Optimal Control For Dysentery diarrhea epidemic Model

In this Section we use the optimal control theory to investigate the behavior of the model system. Our main goal is to vaccinate as many susceptibles as to minimize the number of infected individuals due to diarrhea and the cost of this strategy. The action of this drug can be controlled optimally by applying the maximum principle of Pontryagin. The maximum principle of Pontryagin is a condition such that it can be obtained by completion of optimal control in accordance with the objective of maximizing performance index [9], . The state equation is:

ẋ = f(x(t), u(t), t) dan $x = \begin{bmatrix} S \\ I \\ R \\ T \end{bmatrix}$ he state equation becomes

$$\dot{x} = \begin{bmatrix} S \\ I \\ R \\ T \end{bmatrix}$$ or we have

$$\dot{x} = \begin{bmatrix} (1-þ)A - \beta SI(t) + bV(t) - u_1 S(t) - \mu S(t) \\ I\beta SI(t) + (\alpha+\mu)I(t) - (\alpha+u_2)I(t) - p\emptyset I(t) \\ (\alpha+u_2)I(t) + \tau T(t) - \mu R(t) \\ p\emptyset I(t) - \tau T(t) - \mu I(t) \end{bmatrix}$$

The terminal time T, the problem is to minimize the objective functional J given as:

$$J(u_1, u_2) = \int_0^T \left(I(t) + \frac{a1}{2}u_1^2 + \frac{a2}{2}u_1^2 \right) dt$$

It is also further assumed that there is limitations on the minimum rate of vaccine, treatment, and prevention controls in a given time period T. Hence, a bounded Lebesgue measurable control set is represented as

$U = \{u = (u_1, u_2)\ 0 \le u_i \le u_i \max, i = 1,2\}$

Where φ= (SIR-T) solves equation for the specified control u. In the intervention of controls the solution φ = (SIR-VT) depends on the controls. ai ≥ 0 represents the weights on the benefit and cost. $\frac{a1}{2}u_1^2$ is minimization of cost of vaccine and vaccine rate; $\frac{a2}{2}u_1^2$ is minimization of cost of treatment and treatment rate. The goal is to find an optimal control pair $u* = (u_1*, u_2*)$.

The representation of the optimal controls relies on Pontryagin's maximum principle [12]. To apply this we need to convert the optimal control problem into a problem of minimizing point-wise a Hamiltonian, H, with respect to u. The Hamiltonian associated to our problem is:

$$\begin{aligned} H &= L(x(t), u(t), t) + \\ &\sum_{i=1}^{n} \lambda_i(t)\left(f_i(x(t)), u(t), t\right) \\ &= I(t) + \frac{a1}{2}u_1^2 + \frac{a2}{2}u_1^2 + \\ &\sum_{i=1}^{5} \lambda_i f_i\,(S, I, R, V, T, u) \\ &= I(t) + \frac{a1}{2}u_1^2 + \frac{a2}{2}u_1^2 + \\ &\lambda_1 f_1(S, I, R, T, u) + \\ &\lambda_2 f_2(S, I, R, T, u) + \\ &\lambda_3 f_3(S, I, R, T, u) + \\ &\lambda_4 f_4(S, I, R, T, u) + \lambda_5 f_5(S, I, R, T, u) \end{aligned}$$

$$\begin{aligned} = I(t) + \frac{a1}{2}u_1^2 + \frac{a2}{2}u_1^2 + \lambda_1(1-þ)A - \beta SI(t) \\ + bV(t) - u_1 S(t) - \mu S(t) \\ + \lambda_2 I\beta SI(t) + (\alpha+\mu)I(t) \\ - (\alpha+u_2)I(t) - p\emptyset I(t) \\ + \lambda_3(\alpha+u_2)I(t) + \tau T(t) - \mu R(t) \\ + \lambda_4(þA + u_1 S(t) - bVT(t) - \mu I(t)) \end{aligned}$$

Based on [13] [14], if the control u∗ and the corresponding state φ∗ are an optimal couple, necessarily there exists a non-trivial adjoint vector λ = (λ_1, λ_2, λ_3, λ_4,) satisfying the following equality.

$$\dot{\lambda}_i = -\frac{\partial H}{\partial x_i}, (i = 1,2, \dots n)$$

$$\dot{x}_i = \frac{\partial H}{\partial \lambda_i}, (i = 1,2, \dots, n)$$

This can be rewritten in compact notation as another equation. Next, we check the optimal control and we find that it is indeed a minimum.

So we have

$$u_1^* = \begin{cases} 0, & \frac{(\lambda_1 - \lambda_3)I}{V} \le 0 \\ u_1^*, & 0 < \frac{(\lambda_1 - \lambda_3)I}{V} < u_1\ max \\ u_1\ max, & \frac{(\lambda_1 - \lambda_3)I}{V} \ge u_1\ max \end{cases}$$

$$u_2^* = \begin{cases} 0, & \lambda_3 b \le 0 \\ u_2^*, & 0 < \lambda_3 b < u_2\ max \\ u_2\ max, & \lambda_3 b \ge u_2\ max \end{cases}$$

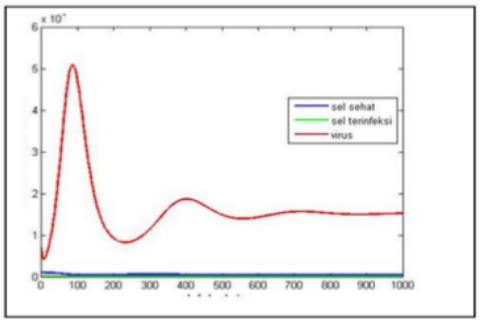

Fig. 2. Simulation model of malaria distribution before being given control

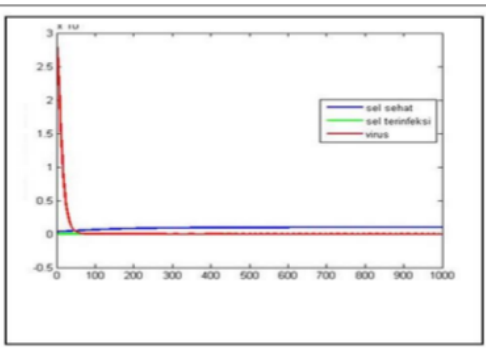

Fig. 3. Simulation model of malaria distribution after being given control

4 Conclusion

Deterministic epidemic model (SIR-T) was considered to gain more insight into the effect treatment of infected individuals on the dynamical spread of diarrhea in a population. This suggests that vaccine should be given as early as possible immediately after birth before the exposure to diarrhea infection treatment to be given must be effective in order to minimize the spread of the disease and the cost.

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