Optimal Control Inventory Stochastic With Production Deteriorating

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Optimal Control Inventory Stochastic With Production Deteriorating

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Abstract. In this paper, we are using optimal control approach to determine the optimal rate in production. Most of the inventory production models deal with a single item. First build the mathematical models inventory stochastic, in this model we also assume that the items are in the same store. The mathematical model of the problem inventory can be deterministic and stochastic models. In this research will be discussed how to model the stochastic as well as how to solve the inventory model   using optimal control techniques. The main tool in the study problems for the necessary optimality conditions in the form of the Pontryagin maximum principle involves the Hamilton function. So we can have the optimal production rate in a production inventory system where items are subject deterioration.
Key words : Inventory production Problem, Stochastic inventory, Control Theory

1. Introduction

Various problems that involve many systems theory, optimal control and some applications. One of them is the inventory problem, the problem is how to manage the change in consumer demand in a finished product. The differential equation model is simplified by using exogenous function, then determined the optimal solution. The next model was developed solution for **t** values towards T. Then use solution with stochastic production deteriorating.

The goal is to determine the average production to minimize some cost function. Related inventory system of optimal control of a deteriorating in the quality of production formed the differential equation, to be determined optimal solutions in the form of the average production so as to minimize some cost function.

2. Theoretical Basic

2.1 The set of convex and convex function

The concept of a convex function underlie some part in the discussion section. The following definitions and theorems related to the set and convex function.

Definition 2.1.1 (Mangasarian) The set $\Gamma \subset \mathbb{R}^n$ called convex set if for to any $x_1, x_2 \in \Gamma$ and for $\lambda \in \mathbb{R}$ such that $0 \leq \lambda \leq 1 \; will \; apply$ $(1-\lambda)(x_1) + \lambda(x_2) \in \Gamma$.

2.2   Nonhomogen Linear Differential Equation and Solution

Definition 2.1.2 (Ross, S.L 1984) Differential equations are equations containing derivatives of one or more dependent variables for   one or more independent variables.

Definition 2.1.3 (Ross, S.L 1984) Order linear differential equations-n, with the dependent variable y, and the independent variable x, can be expressed as follows

$$a_0 x \frac{d^n y}{dx^n} + a_1 x \frac{d^{n-1} y}{dx^{n-1}} + \cdots + a_{n-1} x \frac{dy}{dx} + a_n(x)y = F(x) \tag{2.1}$$

With a_0 not equal to zero. If F is equal to zero then the equation reduces to

$$a_0 x \frac{d^n y}{dx^n} + a_1 x \frac{d^{n-1} y}{dx^{n-1}} + \cdots + a_{n-1} x \frac{dy}{dx} + a_n xy = 0 \tag{2.2}$$

called homogeneous differential equation. For F(x) ≠ 0, referred to as non homogeneous differential equation.

Theorem 2.1.4 (Ross, S.L 1984) Provided y_p a solution to nonhomogeneous linear differential equation (2.1) which does not contain any constants.

2.3 Stochastic Inventory Model

Many phenomena in nature that the occurrence of irregular both in space and time. To model the evolution of a system containing an uncertainty or a system that runs on an unpredictable environment, where the deterministic model is no longer suitable in use for analyze these systems use Stochastic Processes.

Stochastic process is a sequence of events that meet the laws of chance. Stochastic processes are widely used to model the evolution of a system that includes an uncertainty. Or a system that is operating in an unpredictable environment, where the deterministic model is no longer suitable to analyze the system. This model is suitable for the stated model of inventory on a company by the number of average number of requests in which an element of uncertainty. The element of uncertainty is a role model inventory. The company wants sufficient supply to meet customer demand, but production of too many hikes could increase the cost and risk of loss through obsolute or property damage.

2.4 Process Stochastic

The process of stochastic is the set of variables random $\{X(t), t \in T\}$ that describe for $t \in T, X(t)$ dynamic   of a process. The   process   of stochastic is a   collection   random   variable, which is a variable $X(t)$ with distribution   limited. Generally,   index or parameter shows   the time t of the process and the variables   randomly $X(t)$ show the status of the process at the T time. The set index time of the   process called space time (*time   space*) and the   set   of all values of variables random $X(t)$ which may be called the room status (*state space*). If $T = \{1,2,3, \dots\}$ can be calculated then the stochastic is discrete and usually expressed with $X(t)$ where $T = \{t \mid t \geq 0\}$ as   calculated,   then   the stochastic is have   parameter continuous and is   usually   expressed by the notation $\{X(t), t \geq 0\}$. Like wise also the room status, room status is called discrete if finite or not finite can calculated and is called continuous if it contains the interval of the line real.

2.5 Optimal Control

In the following discussion, the problems given in the case of optimal control with state and the end time is known. In other words, the target set S shaped $\{S = x_1(t_1)\}$ in the form (x_1, t_1) with x_1 specially element in R^n and t_1 element at (T$_1$,T$_2$). Given the state system by the end and the end time unknown $\dot{x}(t) = f[x(t), u(t), t]$ with x(t) vector state sized $nx1$, u(t) input vector sized $mx1$, f a vector valued function. Initially given state is x_0 and initially time is t_0. Target set S form (x_1, t_1) with $t_1 \in (T_1, T_2)$ known value and $t_1 > t_0$. Optimal control problem is to find the admissible control u(t) with the initial value (x_0, t_0) and thefinal value (x_1, t_1) that maximizes the objective function

$J(u) = \int_{t_0}^{t_1} L(x(t), u(t), t) \, dt$. To solve the problems mentioned above optimal control, first determined necessary condition for optimal control are met.

3. Discussion of problems

Many company manufacturing using production inventory system are being made to regulate changes in consumer demand in a finished product. So the consumer and customer demand can be met, but also the production of goods is not in decline. The slump in production inventory may occur due to spoilage or damage to goods inventory of production for a certain period.

For that the company must make a plan so that products finished goods in the warehouse did not fit in decline before the booked by consumers and can manage inventories so as not to delay requests made by consumers. This has caused some problems, in addition to ware housing problems that certainly will add to the cost of storage also makes good planning so that the items are in storage until the decline. So the optimization problem that arises is how to balance between the level of production of goods in order to meet the demands of customers and consumers, but did not issue a production inventory storage costs and huge slump. Then some form of illustrative model used is a constant demand and linear request.

3.1. Matematic modelling

Estimated optimal state for linear systems with noise and noise measurement. Also the white noise process will be defined to obtain Kalman-Bucy filter for continuous time system. Next will be introduced the possibility of controlling the system governed by Ito in stochastic differential equations. In other words variable controls will be introduced into nonlinear versions of the previous equation. This results in the formulation of stochastic optimal control problems. It should be noted that for such problems, the principle of separation is not generally applicable. Therefore, to simplify the treatment, it is often assumed that state variables can be observed, in the sense that state variables can be directly measured.

Furthermore, much of the literature on this issue uses dynamic programming or Hamilton-Jacobi-Bellman frameworks to determine the maximum stochastic principle. The following will be given the formulation of the stochastic optimal control problem by providing a brief, informal development of the Hamilton-Jacobi-Bellman (HJB) solution form for the solution of the problem equation.

3.2 Optimal Control of Stochastic Production Inventory with deteriorating

The following will introduce the notation and symbols used in the stochastic production inventory model. In this research used generalization of model obtained by Sethi and Thompson (2000). The assumption is used by assuming the plant produces a single homogeneous single and has a finished product in the warehouse. To illustrate the state of the following model in defining parameters and variables in the system, is as follows:

X_t : inventory level at time t,
U_t : production rate at time t,
$S(t)$: the level of demand at time t,
T : long period of planning,
X : factory inventory level k,
$\hat{u}$: factory production objectives,
x_0 : initial inventory level,
h : cost coefficient Inventory storage,
c : coefficient production cost,
B : residual value per unit of inventory when T,
z_t : standard Wiener process,
σ : diffusion coefficient,
$\emptyset(t)$: deteriorating.

The factory inventory level X is the safety stock level that the company wants to keep the amount of inventory safe in storage. Also, U-level factory production level is the most efficient level desire to run the plant. Using the above notation, you can describe the model condition. Equation

$$X(t) = u(t) - S(t) - \emptyset(t), \quad x(0) = x_0, \tag{3.1}$$

First is the model flow equation, Which states that inventory levels at time t increase with production and decrease with demand. Then equation (3.1) can be generalized to a stochastic model. By making the inventory level assumption x_t follows the I$_{to}$ Differential Stochastic equation as follows :

$$dX_t = (U_t - S_t - \emptyset)dt + \sigma dz_t, \quad X_0 = x_0, \tag{3.2}$$

By using process can be expressed as $w(t)dt$, where $w(t)$ is as white noise process. This process can be interpreted as the sale of random returns or random decomposition in nature. In this model there is no limitation of production rate to be negative. This means that permitted disposal ($u_t < 0$). This states the conditions under which a disposal is not required. Furthermore the inventory level is allowed to become negative.

The solution of this problem will be done by using Hamilton-Jacobi modified The principle generated by the Hamilton-Jacobi-Bellman equation is called that satisfied with a certain value function. Now, the problem is to minimize the total expected cost as determined by the following functional

$$J(u) = E\left\{\int_0^T c(U_t - \hat{u})^2 + h(X_t - \hat{x})^2 dt + BXt\right\} \tag{3.3}$$

Assume that V (x, t) denotes the minimum value of the total expected cost from t to T with $X_t = x$ using the optimal policy from t to T. Then the function is given by:

$$V(x,t) = \min_{U_t} E\left\{\int_t^T c(U_t - \hat{u})^2 + h(X_t - \hat{x})^2 dt + BXt\right\} \tag{3.4}$$

Therefore the function value of $V(x,t)$ must satisfy the Hamilton-Jacobi-Bellman (HJB) equation

$$0 = \max_z \left\{-c(U_t - \hat{u})^2 - h(x - \hat{x})^2 + \frac{\partial V}{\partial t} + \frac{\partial V}{\partial x}(u - S) + \frac{1}{2}\sigma^2 \frac{\partial^2 V}{\partial x^2}\right\} \tag{3.5}$$

With boundary conditions $V(x,t) = Bx$ \hfill (3.6)

Now to maximize the expression with respect to u by taking its derivative with respect to u and setting it to zero. Procedure of this result

$$\frac{\partial V}{\partial x} - 2c(u - \hat{u}) = 0 \tag{3.7}$$

Therefore, the optimal production rate that minimizes the total cost can be expressed as a function of the function by determining the value of the known equation at the moment in the form of:

$$u(x,t) = \frac{1}{2c}\frac{\partial V}{\partial x} + \hat{u} \tag{3.8}$$

With substitution (3.8) to (3.5) we obtain an equation

$$0 = -\left[\frac{1}{4c}\left(\frac{\partial V}{\partial x}\right)^2 + h(x - \hat{x})^2\right] + \frac{\partial V}{\partial t} + \frac{\partial V}{\partial x}\left[\frac{1}{2c}\left(\frac{\partial V}{\partial x}\right) + \hat{u} - S(t)\right] + \frac{1}{2}\sigma^2 \frac{\partial^2 V}{\partial x^2} \tag{3.9}$$

To solve the nonlinear partial differential equation (3.9), let's use the assumption that the solution takes the following form

$$u(x,t) = max\left[0, \frac{\partial V}{\partial x} + \hat{u}\right] \tag{3.10}$$

3.3 The Hamilton-Jacobi-Bellman Equation Model Solution

In this section we will find solutions of stochastic production planning problems with different levels of demand. To solve the nonlinear partial differential equation (3.9), let's use the assumption that the solution takes the following form

$$V(x,t) = Q(t)x^2 + R(t)x + M(t) \tag{3.11}$$

Then $\quad \dfrac{\partial V}{\partial x} = 2Qx + R$ \hfill (3.12)

$$\frac{\partial^2 V}{\partial x^2} = 2Q \tag{3.13}$$

$$\frac{\partial V}{\partial t} = \dot{Q}x^2 + \dot{R}x + \dot{M} \tag{3.14}$$

At which point denotes differentiation with time. Substituting from equation (3.9) to equation (3.11) will give results

$$\frac{1}{4c}[2Qx + R]^2 - h[x^2 - 2\hat{x}x + \hat{x}^2] + [\dot{Q}x^2 + \dot{R}x + \dot{M}] + [2Qx + R](\hat{u} - S) + \sigma^2 Q = 0 \tag{3.15}$$

Equation (3.15) should apply to any value of x, we get the following system of ordinary nonlinear differential equations:

$$c\dot{Q} = ch - Q^2 \tag{3.16}$$
$$c\dot{R} + QR = -2ch\hat{x} - 2cQ(\hat{u} - S) \tag{3.17}$$
$$4c\dot{M} = 4ch\hat{x}^2 - R^2 - 4c(\hat{u} - S)R + c\sigma^2 Q \tag{3.18}$$

Furthermore, these nonlinear systems will be solved for different cases of demand level with

$$Q(T) = 0, R(T) = B, M(T) = 0. \tag{3.19}$$

By changing time t into:

$$\tau = e^2\sqrt{h/c} \; (t\text{-}T). \tag{3.20}$$

Then, the value will be obtained $\quad \frac{\partial}{\partial t} = 2\sqrt{\frac{h}{c}} \; r \; \frac{\partial}{\partial r},$

And a certain time will be obtained two different common cases for the level of demand

$$\frac{t}{t} - e^{-2} = 0 \Rightarrow r = e^{-2} \sqrt{\frac{h}{c}} T, \text{ and } t = T \Rightarrow r = 1, \text{ Means will be obtained value}$$

$$r \in \left[e^{-2} \sqrt{\frac{h}{c}} T, 1 \right]. \tag{3.21}$$

To solve equation (3.16) can be used $\frac{c\dot{Q}}{(ch-Q^2)}$ So it will be obtained

$$Q(\tau) = \frac{\sqrt{ch} \, (\tau-1)}{(\tau+1)} \tag{3.22}$$

Next will be discussed two general cases to determine the average demand. The first case is the case where the constant demand level is S (t) = S_0 = const. In this case the optimal level of production is provided by:

$$u(x,t) = \frac{1}{c}\left[\left(\frac{\sqrt{ch}\,(\tau-1)}{(\tau+1)}\right)x - c(\bar{u} - S_0) + [B + 2c(\bar{u} - S_0)]\frac{\sqrt{\tau}}{(\tau+1)} - \hat{x}\sqrt{ch}\left(\frac{\tau-1}{\tau+1}\right)\right] + \bar{u} \tag{3.23}$$

The functions R (τ) and M (τ) will be obtained by:

$$R(\tau) = -c(\bar{u} - S_0) + 2[B + 2c(\bar{u} - S_0)]\frac{\sqrt{\tau}}{(\tau+1)} - 2\hat{x}\sqrt{ch}\left(\frac{\tau-1}{\tau+1}\right) \tag{3.24}$$

$$M(\tau) = \int\left\{\frac{1}{2\tau}\left[\sqrt{ch}\hat{x}^2 - \frac{R^2}{4\sqrt{ch}} - \sqrt{\frac{c}{h}}(\bar{u} - S_0)R - \sqrt{\frac{c}{h}}\sigma^2 Q\right]\right\}d\tau + M_0 \tag{3.25}$$

In order to find the expected level of inventory, the state equation (3.2) can be solved by taking the mean in relation to the state of the variable x and substituting it with the optimal production rate of (3.23) to obtain:

$$\dot{E}(x) = \frac{Q}{c}E(x) = \frac{1}{c}\left\{[B + 2c(\bar{u} - S_0)]\frac{\sqrt{\tau}}{(\tau + 1)} - \hat{x}\sqrt{ch}\left(\frac{\tau-1}{\tau+1}\right)\right\}$$

$$E(x) = \hat{x} - \frac{B+2c(\bar{u}-S_0)}{2\sqrt{ch}\sqrt{\tau}} + \left[x_0 - \hat{x} + \frac{B+2(\bar{u}-S_0)}{2\sqrt{ch}\sqrt{r_0}}\right]\frac{\sqrt{r_0}}{1+r_0}\left(\frac{r+1}{\sqrt{r}}\right) \tag{3.26}$$

Some special cases can be obtained from this, for example when the level of demand. Constant and equal to U level production objectives. In this case the expected supply level will be provided by the following functions:

$$E(x) = \hat{x} + (x_0 - \hat{x})\left[\frac{\sqrt{r_0}\,(r+1)}{(1+r_0)\sqrt{r}}\right] - \frac{B}{2\sqrt{ch}}\left[1 - \left(\frac{\tau+1}{r_0+1}\right)\right]\frac{1}{\sqrt{r}} \tag{3.27}$$

So the total expectation cost of $E[V(x, T)]$ will be obtained in the form

$$E[V((x, T), T)] = B\left\{\hat{x} + (x_0 - \hat{x})\left[\frac{\sqrt{r_0}\,(r+1)}{(1+r_0)\sqrt{r}}\right] - \frac{B}{2\sqrt{ch}}\left[\frac{1}{\sqrt{r}} - \left(\frac{(\tau+1)}{r_0+1(\sqrt{r})}\right)\right]\right\} \tag{3.28}$$

The second case is the case where the level of time request of the level varies, with the form of the obtained value being S (t) = u-2τ√τ / τ-1. The optimal level of production in this case is given by:

$$u(x,t) = \frac{1}{c}\left[\left(\frac{\sqrt{ch}\,(r-1)}{r+1}\right)x + \frac{(c+b)\sqrt{r}}{1+r} - \sqrt{\frac{q_0}{h}}\left(\sqrt{ch}\,\frac{r\sqrt{r}}{1+r} + h\hat{x}\frac{(r-1)}{r+1}\right)\right] + \bar{u} \tag{3.29}$$

In the second case will be obtained, the function R (τ) will be given with

$$R(\tau) = \frac{2(c+b)\sqrt{r}}{\tau+1} - 2\sqrt{\frac{c}{h}}\left(\sqrt{ch}\,\frac{r\sqrt{r}}{1+r}\left(\frac{\tau-1}{\tau+1}\right)\right) \tag{3.30}$$

The following calculation values will be obtained

$$E(x) = \frac{1}{2}\sqrt{\frac{c}{h}}\left[\left(\frac{c+B}{c}\right)\frac{\sqrt{r}}{1+r}\ln\tau - 2\sqrt{\frac{c}{h}}\,\hat{x} + \frac{\sqrt{r}}{1+r}\ln(\tau-1)^2\right] + E_0\frac{\sqrt{r}}{1+r} \tag{3.31}$$

Where $E_0 = (x_0 + \hat{x})\left(e^{\sqrt{\frac{h}{c}}T} + e^{-\sqrt{\frac{h}{c}}T}\right) - \left(\frac{c+B}{c}\right)T + \frac{1}{2}\sqrt{\frac{c}{h}}\ln\left(e - 2\sqrt{\frac{h}{c}}T - 1\right)^2$ (3.32)

Then the inventory level of inventory will be provided with

$$E(x) = \frac{1}{2}\sqrt{\frac{c}{h}}\left[\left(\frac{c+B}{c}\right)\frac{\sqrt{r}}{1+r}\ln\tau - 2\sqrt{\frac{c}{h}}\,\hat{x} + \frac{\sqrt{r}}{1+r}\ln(\tau-1)^2 + \frac{\sqrt{r}}{1+r}\right](x_0+\hat{x})\left(e^{\sqrt{\frac{h}{c}}T} + e^{-\sqrt{\frac{h}{c}}T}\right) -$$

$$\left(\frac{c+B}{c}\right)T + \frac{1}{2}\sqrt{\frac{c}{h}}\ln\left(e - 2\sqrt{\frac{h}{c}}T - 1\right)^2$$

$$(3.33)$$

4.Conclusions and Suggestions

From the discussion obtained some conclusions :

Inventory models of both raw material inventory and inventory deterministic and stochastic can be solved by control theory. Proper production arrangements are crucial in order to manage the quantities of the goods produced in a level that balances demand and risk and Optimal control can be used to complete the Inventory model with Stochastic Production Model. For future researchers, it is advisable to research the application of Theory of Control into other areas of research operations.

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