

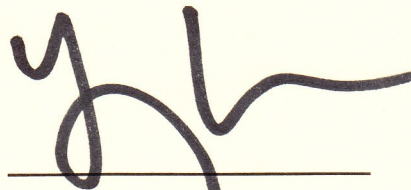
Graduate Student Conference
Certificate of Participation



This is to certify that

Muhammad Ahsar Karim

Participated in the Oral Presentations at the
Graduate Student Conference 2017
held on 28 October 2017 at the
National Institute of Education, Singapore



Ast/P Adrian Kee

Assistant Dean, Higher Degrees by Research
Office of Graduate Studies & Professional Learning
National Institute of Education, Nanyang Technological University

Acceptance of abstract submission for Graduate Student Conference 2017

NIE Graduate Students Conference (GPL) <niegsc@nie.edu.sg>
To: "m_ahsar@unlam.ac.id" <m_ahsar@unlam.ac.id>

Sat, Sep 23, 2017 at 5:34 PM

Dear Muhammad Ahsar Karim,

1. We have accepted your abstract submission entitled "**A Comparison of Numerical Performances between Classical and Extended Runge-Kutta Methods for Parameter Estimations of Periodic Type Fuzzy Differential Equations**" for **Oral Presentation** at the Graduate Student Conference 2017.

2. Here are some Presentation Guidelines for Oral Presentation.

- a) Oral paper presentations will be held in parallel sessions
- b) Each parallel session shall consist of a number of paper presentations
- c) You will be allocated 15 minutes to present your paper, and 5 minutes for Questions and Answers
- d) The usual projection system will be available for your use at the presentation venues
- e) The actual presentation timeslot has not been confirmed. We will inform you in due course.

3. Directions to the Graduate Student Conference 2017 (for overseas presenters)

- a) The Graduate Student Conference 2017 will be held in the National Institute of Education (Singapore), an institute in the Nanyang Technological University (Singapore).
- b) For the map of Nanyang Technological University, please see <http://maps.ntu.edu.sg/maps>
- c) -For the map of National Institute of Education (Singapore), please see <http://www.nie.edu.sg/about-us/visit-us>

4. Should you have any enquiries, please email us at niegsc@nie.edu.sg

5. See you at the Graduate Student Conference 2017 on 27-28 October 2017.

Regards,
Organising Committee
Graduate Student Conference 2017
National Institute of Education, Singapore.

A COMPARISON OF NUMERICAL PERFORMANCES BETWEEN CLASSICAL AND EXTENDED RUNGE-KUTTA METHODS FOR PARAMETER ESTIMATIONS OF PERIODIC TYPE FUZZY DIFFERENTIAL EQUATIONS

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Motivations

Modeling in complex systems is often faced with challenges in terms of **measurement uncertainty**
*This is possibly either due to **limitations of available data**, **environmental** or **demographic changes***

← Uncertainty Theory

↓
Initial Value Problem with **Periodic Behavior**
(Harmonic Oscillator Model)

←
• Probabilistic
• Linguistic

↓
Periodic Type **Fuzzy Differential Equations (FDEs)**
(Fuzzy Initial Value Problem)

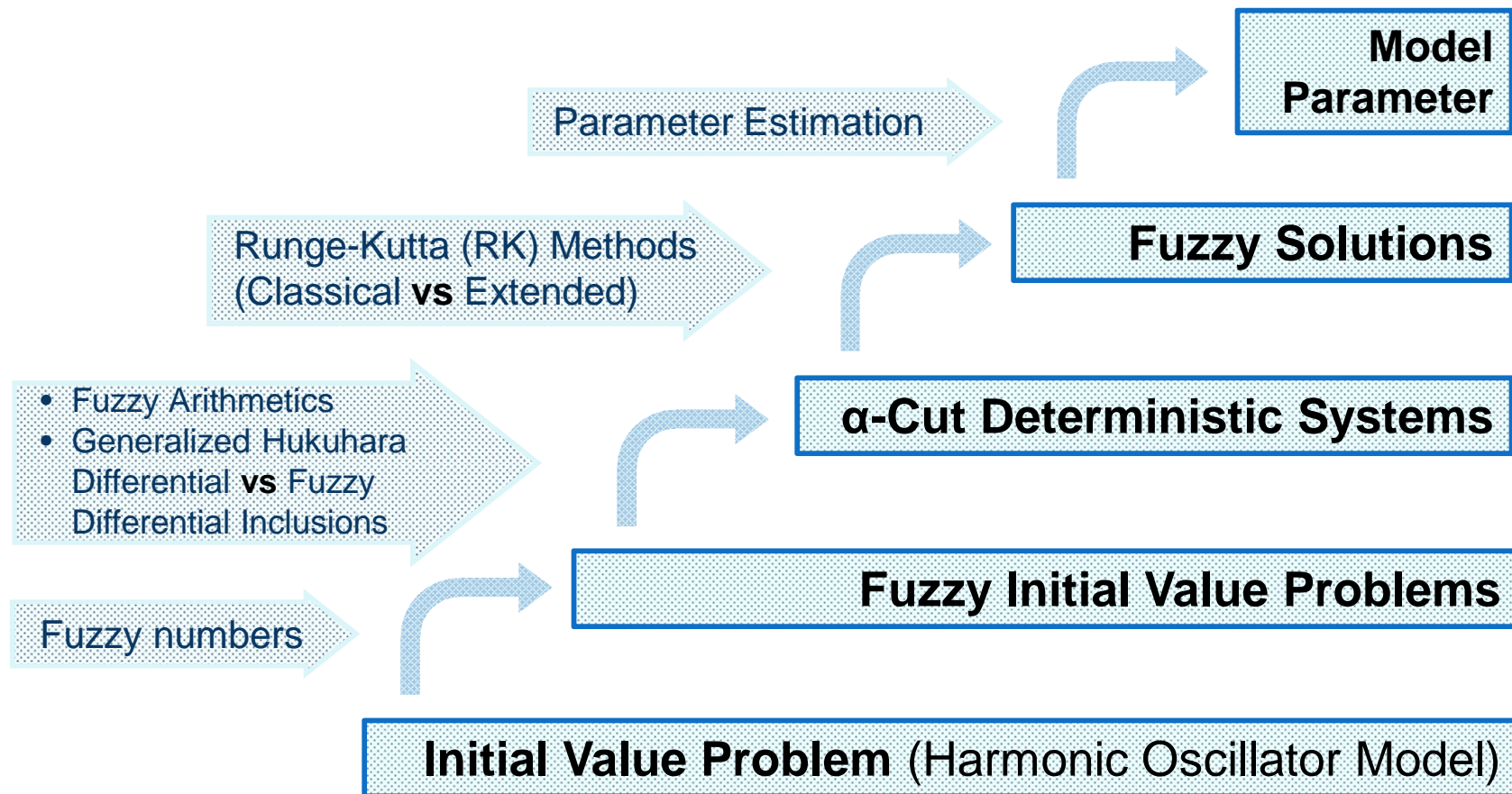
← Fuzzy Theory (Linguistic)

↓
Solving the FDEs

←
• Fuzzy Arithmetics
• Fuzzy Differential Types
• Solving Methods

↓
Parameter Estimation

Methodology



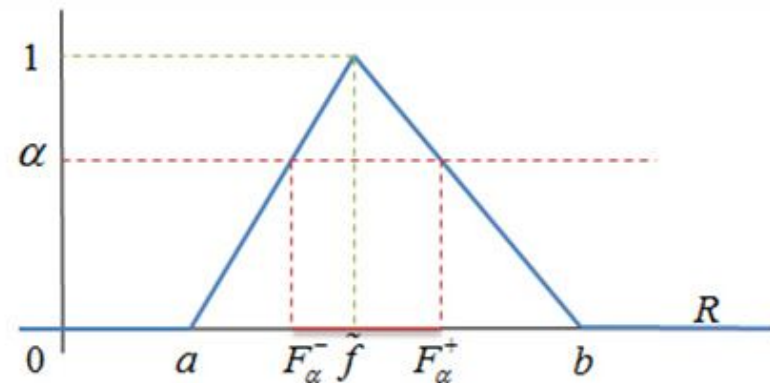
Membership Function

Some of the basic concepts of fuzzy theory, such as fuzzy subset, α -cut, fuzzy number, fuzzy difference, Hukuhara differentiable, and Seikkala differentiable can be found in [6,9,10].

An illustration:

Given a triangular membership function F (one of the forms of fuzzy number) below. The fuzzy number F usually called by “**around f** ”.

$$\text{trimf}_F(x, [a, \bar{f}, b]) = \begin{cases} \frac{x-a}{\bar{f}-a} & ; a \leq x < \bar{f} \\ \frac{b-x}{b-\bar{f}} & ; \bar{f} \leq x < b \\ 0 & ; x \text{ others} \end{cases}$$



with $a, \bar{f}, b \in R$ and $a < \bar{f} < b$.

The α -cut of F shortened by $[F]^\alpha = [F_\alpha^-, F_\alpha^+], \alpha \in [0, 1]$.

Arithmetic of fuzzy numbers

Let A and B be fuzzy numbers with α -cuts $[A]^\alpha = [A_\alpha^-, A_\alpha^+]$ and $[B]^\alpha = [B_\alpha^-, B_\alpha^+]$, respectively, and a real number δ .

(a) The sum and the difference of $[A]^\alpha$ and $[B]^\alpha$:

$$[A + B]^\alpha = [A]^\alpha + [B]^\alpha = [A_\alpha^- + B_\alpha^-, A_\alpha^+ + B_\alpha^+] \text{ and}$$

$$[A - B]^\alpha = [A]^\alpha - [B]^\alpha = [A_\alpha^- - B_\alpha^+, A_\alpha^+ - B_\alpha^-].$$

(b) The multiplication of $[A]^\alpha$ by δ :

$$[\delta A]^\alpha = \delta[A]^\alpha = \delta[A_\alpha^-, A_\alpha^+] = \begin{cases} [\delta A_\alpha^-, \delta A_\alpha^+]; & \delta \geq 0 \\ [\delta A_\alpha^+, \delta A_\alpha^-]; & \delta < 0 \end{cases}$$

(c) The multiplications of $[A]^\alpha$ and $[B]^\alpha$:

$$[A \cdot B]^\alpha = [A]^\alpha \cdot [B]^\alpha = [\min P, \max P];$$

$$P = \{A_\alpha^- B_\alpha^-, A_\alpha^- B_\alpha^+, A_\alpha^+ B_\alpha^-, A_\alpha^+ B_\alpha^+\}$$

(d) The division of $[A]^\alpha$ by $[B]^\alpha$, if $0 \notin \text{supp}(B)$:

$$[A / B]^\alpha = [A]^\alpha / [B]^\alpha = [A_\alpha^-, A_\alpha^+] \cdot [1 / B_\alpha^+, 1 / B_\alpha^-]$$

Fuzzy Differential Equations

- ❖ Let R_F be the set of fuzzy numbers, $F, G: (a, b) \rightarrow R_F$, $(a, b) \subseteq R$.
If F and G are **Seikkala differentiable**, then
 $(F + G)' = F' + G'$ and $(kF)' = kF'$, $\forall k \in R$.
- ❖ Let $\mathfrak{F}_\varphi(R)$ be the family of all the fuzzy numbers on R ,
 $F: (a, b) \rightarrow \mathfrak{F}_\varphi(R)$ and $F(x) = (f_\alpha^-(x), f_\alpha^+(x))$.
 - (a) If F is **Hukuhara Differential (HD)**, then $F' = (f_\alpha^{-\prime}, f_\alpha^{+\prime})$.
 - (b) If F is **Generalized HD (GHD)**, then $F' = (f_\alpha^{+\prime}, f_\alpha^{-\prime})$.

A HD function is also a Seikkala differentiable.

❖ A Fuzzy Differential Inclusion (FDI)

(i) defined as

$$y'(t) \in F(t, y(t)), y(0) \in \tilde{y}_0$$

(ii) interpreted as

$$y'(t) \in [F(t, y(t))]^\alpha, y(0) \in [\tilde{y}_0]^\alpha; \alpha \in [0, 1],$$

where $[F]^\alpha: [0, T] \times R^n \rightarrow \mathfrak{F}_\varphi(R)$ and $[\tilde{y}_0]^\alpha \in \mathfrak{F}_\varphi(R)$.

If F is continuous and bounded, then all the solutions to FDI are defined and are the α -cuts of the fuzzy solution.

Runge-Kutta (RK) Methods

Let the system of ordinary differential equations:

$$y'(x) = f(x, y(x))$$

Classical RK Method

The general form of the main function:

$$y_{i+1} = y_i + (a_1 k_1 + a_2 k_2 + \dots + a_n k_n) h$$

with the evaluation functions k_i are:

$$k_1 = f(x_i, y_i)$$

$$k_2 = f(x_i + p_1 h, y_i + q_{11} k_1 h)$$

$$k_3 = f(x_i + p_2 h, y_i + q_{21} k_1 h + q_{22} k_2 h)$$

...

$$k_n = f(x_i + p_{n-1} h, y_i + q_{n-1,1} k_1 h +$$

$$q_{n-1,2} k_2 h + \dots + q_{n-1,n-1} k_{n-1} h)$$

where a_i , p_i and $q_{i,j}$ are constants.

Extended RK Method

The general form of the main function:

$$y_{n+1} = y_n + \sum_{i=1}^m (h b_i k_{i1} + h^2 c_i k_{i2})$$

with the evaluation functions k_{i1} and k_{i2} are:

$$k_{i1} = f\left(x_n + \bar{c}_i h, y_n + h \sum_{s=1}^{i-1} a_{is} k_{s1}\right)$$

$$k_{i2} = f'\left(x_n + \bar{c}_i h, y_n + h \sum_{s=1}^{i-1} a_{is} k_{s1}\right)$$

where b_i , c_i , \bar{c}_i and a_{is} are constants.

The f' is approximated by **forward difference method**.

Comparison of Fourth-order RK Methods

Examples of models with **available analytical solutions**:

Equations	Exact solution
(1). $y' = \frac{1}{80}(20y - y^2); y(0) = 1, x = [0,10]$	$y = \frac{20}{1 + 19\exp(-x/4)}$
(2). $y' = y + x^2 + 1; y(0) = 1, x = [0,10]$	$y = 4e^x - x^2 - 2x - 3$

$$Error_{\max} = \max\{|Num_i - Analitic_i|, i = 1, 2, \dots, n\}$$

Eq. 1 (h)	$Error_{\max}$	
	Classical RK4	Extended RK4
0.10	1.64e-08	1.72e-09
0.05	1.03e-09	1.09e-10

Eq. 2 (h)	$Error_{\max}$	
	Classical RK4	Extended RK4
0.10	6.32e-01	9.21e-02
0.05	4.12e-02	5.75e-03

Comparison of Fourth-order RK Methods

Examples of models with periodical type fuzzy differential equations.

Let Van der Pol model in the form of FIVP:

$$\begin{aligned}\tilde{y}_1' &= \tilde{y}_2, \\ \tilde{y}_2' &= \lambda \tilde{y}_2 (1 - \tilde{y}_1^2) - \tilde{y}_1, \\ \tilde{y}_1(0) &= \tilde{y}_{1_0}, \tilde{y}_2(0) = \tilde{y}_{2_0} \in \mathfrak{F}_\varphi(R), \quad \lambda \in R\end{aligned}$$

α -cuts of state variables $[\tilde{y}_1]^\alpha = [y_{1\alpha}^-, y_{1\alpha}^+]$ and $[\tilde{y}_2]^\alpha = [y_{2\alpha}^-, y_{2\alpha}^+]$,

initial conditions:

$$\tilde{y}_{1_0} = A = \text{trimf}(x, [0, 1, 2]) \quad \text{and} \quad \tilde{y}_{2_0} = B = \text{trimf}(x, [-1, 0, 1]),$$

with α -cuts:

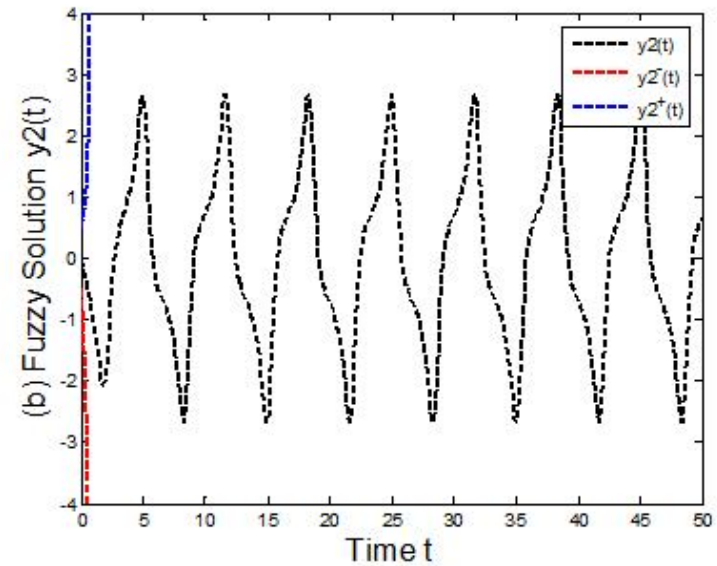
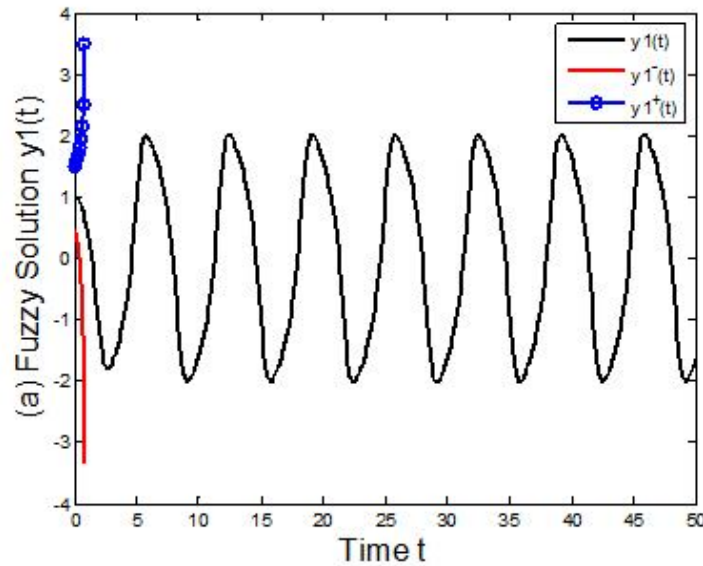
$$\begin{aligned}[\tilde{y}_{1_0}]^\alpha &= [y_{1_0\alpha}^-, y_{1_0\alpha}^+] = [0.5, 1.5], \\ [\tilde{y}_{2_0}]^\alpha &= [y_{2_0\alpha}^-, y_{2_0\alpha}^+] = [-0.5, 0.5]\end{aligned}$$

for $\alpha = 0.5$.

Analytical solution is **not available** of the FIVP.

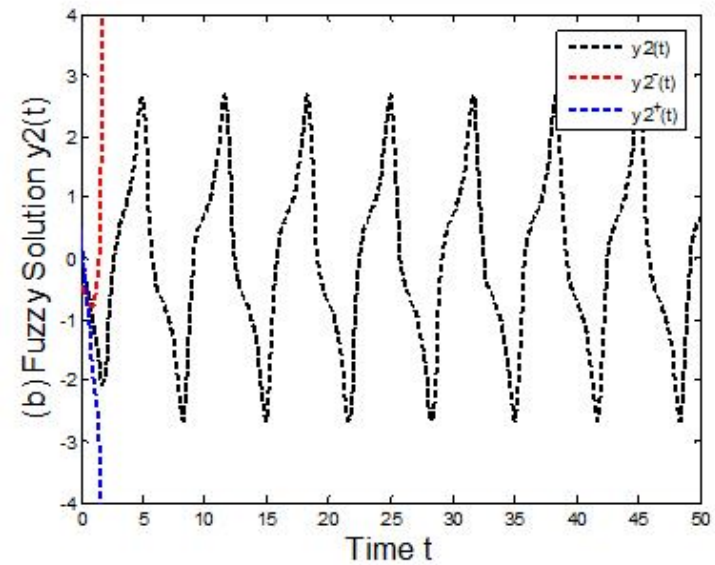
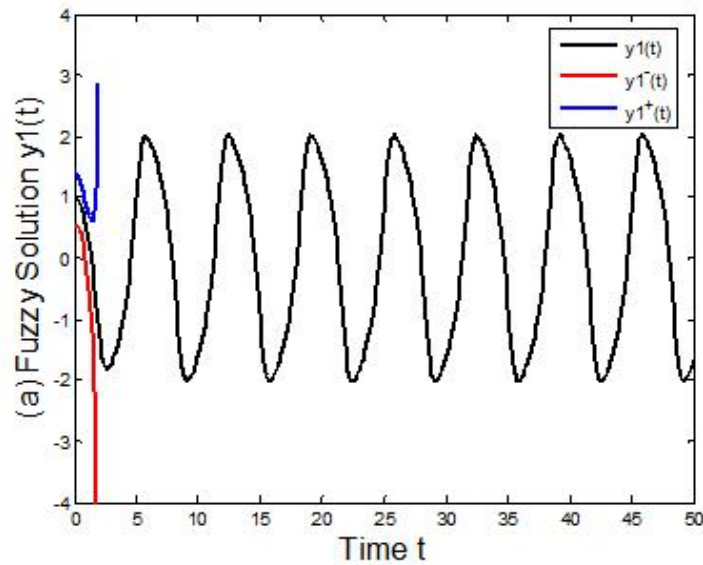
Comparison of Fourth-order RK Methods

❖ Numerical solution using HD concept:



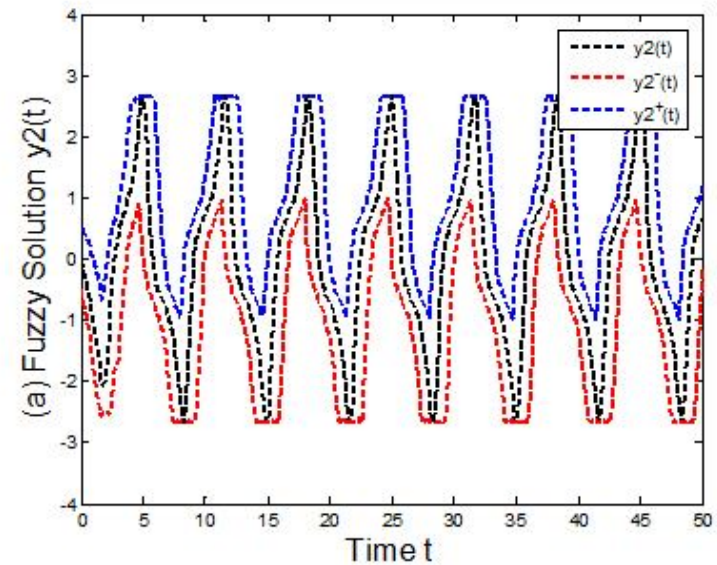
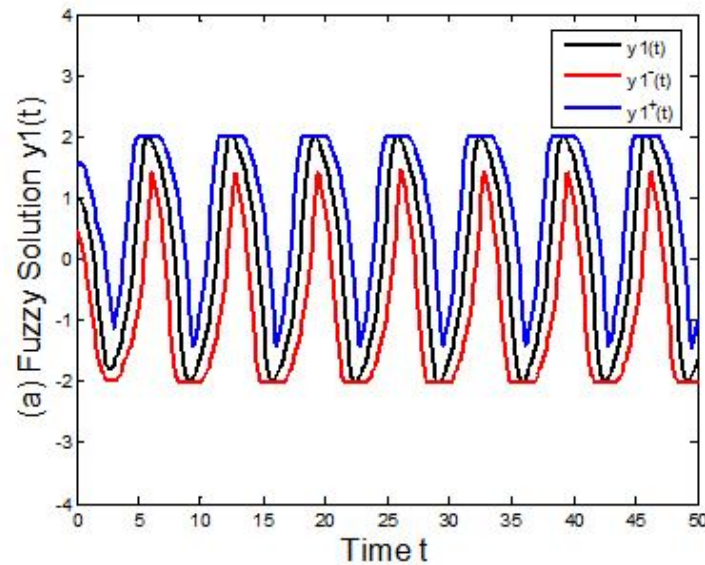
Comparison of Fourth-order RK Methods

❖ Numerical solution using GHD concept:



Comparison of Fourth-order RK Methods

- ❖ Numerical solutions using FDI concept:



Performing comparison of methods, it takes **real data**. But the experiments of this problem are **still going on**.

In this presentation we only show the **parameter estimation** process of FIVP Van der Pol model.

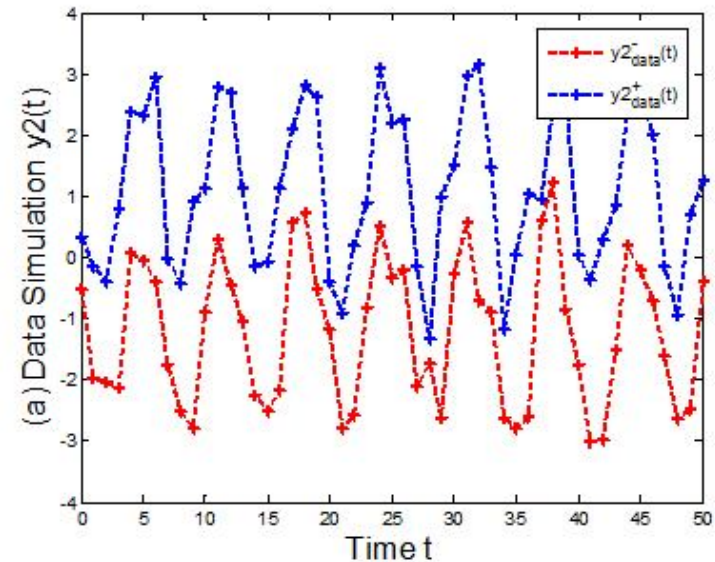
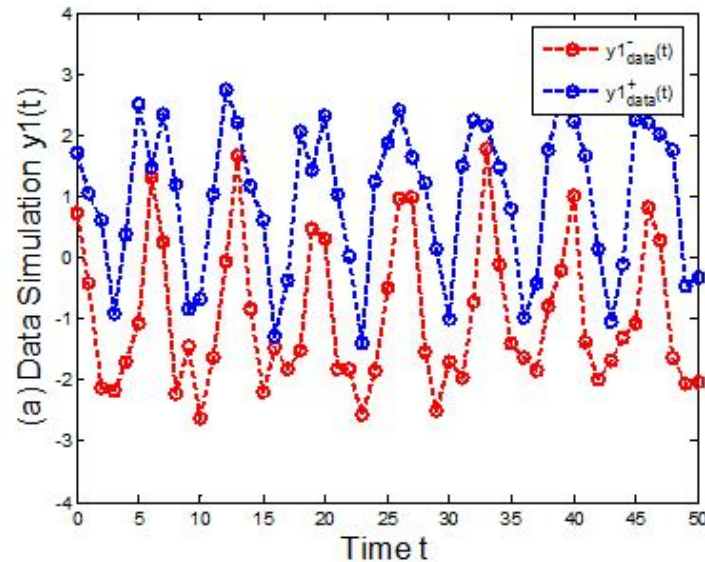
Parameter Estimation

Let the α -cut of **data simulation**:

$$[\tilde{y}_{1\text{data}}(t)]^\alpha = [y_{1\text{data}}^-(t, \alpha), y_{1\text{data}}^+(t, \alpha)]$$

$$[\tilde{y}_{2\text{data}}(t)]^\alpha = [y_{2\text{data}}^-(t, \alpha), y_{2\text{data}}^+(t, \alpha)]$$

for fixed $\alpha \in [0,1]$ and $t = [0,10]$.



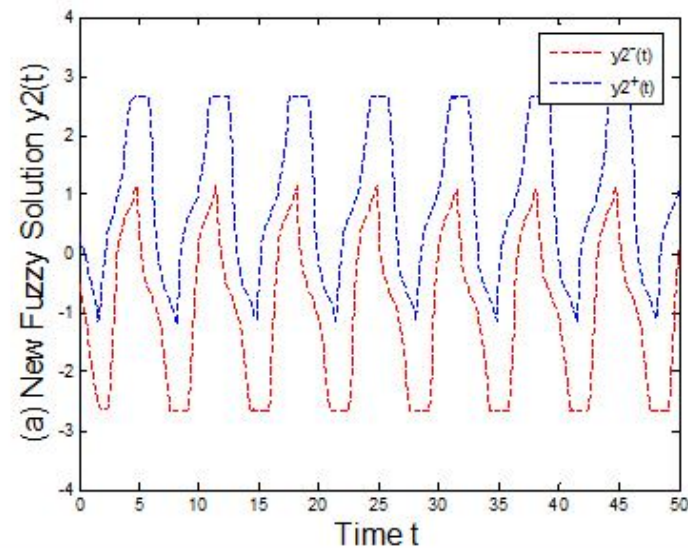
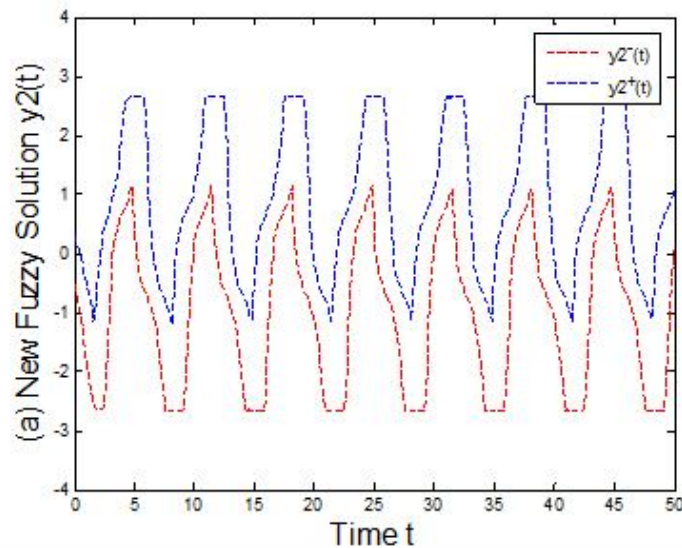
Parameter Estimation

Applying **FDI** concept & **LSQ-Nonlin** method, then optimizing objective function:

$$\min_k \|F(k)\|_2^2 = \min_k \frac{1}{M} \left(\sum_{t=0}^{N-1} (y_{1\alpha t}^- - y_{1data}^- \alpha t)^2 + \sum_{t=0}^{N-1} (y_{1\alpha t}^+ - y_{1data}^+ \alpha t)^2 + \sum_{t=0}^{N-1} (y_{2\alpha t}^- - y_{2data}^- \alpha t)^2 + \sum_{t=0}^{N-1} (y_{2\alpha t}^+ - y_{2data}^+ \alpha t)^2 \right)$$

with $M = 4N$ and $k \in [0, 2]$.

Producing the parameter $k = 0.99999996$ & resnorm = 0.22



Conclusions

- Extended Runge Kutta method is significantly better than Classical Runge Kutta method in our some studies. But in the case of periodical type dynamical systems, it is need to studies more intensively.
- In this paper we showed how to choose the appropriate fuzzy method to capture uncertainty for a system having periodical type dynamical system, i.e. Van der Pol model in the form of FIVP. Two method, namely Hd and gHd concepts were not able to capture the oscillations. In contrast, the method using FDI concept was able to capture the oscillations and maintained the uncertainty of the solutions. This led us to apply the FDI concept to estimate the parameters of the FIVP.



Thank You

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